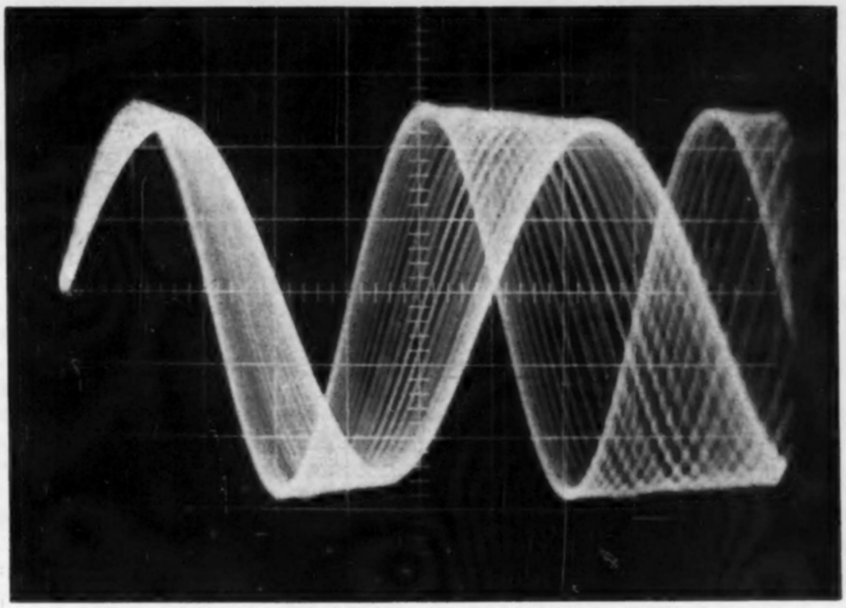

Oscillator output waveform, with a 10-cps control voltage varying the frequency ± 20 percent from the center frequency of 1,000 cps

VOLTAGE CONTROLLED

Wide-Range Oscillator

Small-signal a-c resistance of junction diode is related to reciprocal of junction current over a two-decade range. Using this characteristic in a two-section R-C shift network provides voltage-controlled phase shift

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THE PROBLEM in the design of a wide-range voltage-controlled sinusoidal oscillator is to find a voltage or current sensitive impedance that will vary predictably over a wide range. Vacuum tubes[1, 2] and transistors[3] have been considered as control impedances in voltage-controlled oscillators. Semiconductor devices have small-signal impedances that vary over a considerable range with bias conditions.

A control element was chosen and incorporated in a two-section phase-shift network to provide a voltage-controlled phase shift. This controllable phase shift, when coupled with an amplifier and an automatic gain control, constitutes a constant-amplitude voltage-controlled oscillator with a frequency range of over two decades.

The quest for a voltage or current-sensitive impedance in semiconductor devices was pursued on a theoretical basis by examining the small-signal equivalent impedance of *p-n* junctions. A complete analysis of the equivalent impedance is complex, but for a range of bias currents of a few decades and with applied frequencies in the audio range, the approximate equivalent circuit can be considered to consist of a capacitor and a resistor in parallel.

The small-signal *p-n* diode resistance is derived from the ideal diode equation.[4] It is given by

$$r_{ac} = \frac{kT/e}{I_o + I_i} \qquad (1)$$

Thus for I_i much greater than I_o, the small-signal resistance is proportional to the reciprocal of the junction current. This holds true only for the ideal diode; with an actual diode the ohmic resistance of the semiconductor and leads must be taken into account. For the audio range, the capacitive reactance of both the transition and diffusion capacitance of the diode is much greater than signal resistance at a given bias level, hence the diode is almost purely resistive.

The effect of changes in junction-temperature on the equivalent small-signal resistance depends on the applied bias. For constant-current bias with I_i much greater than I_o and at a temperature of 300 Kelvin, the relationship

$$r_{ac} = \frac{kT/e}{I_i} \qquad (2)$$

shows that the resistance changes

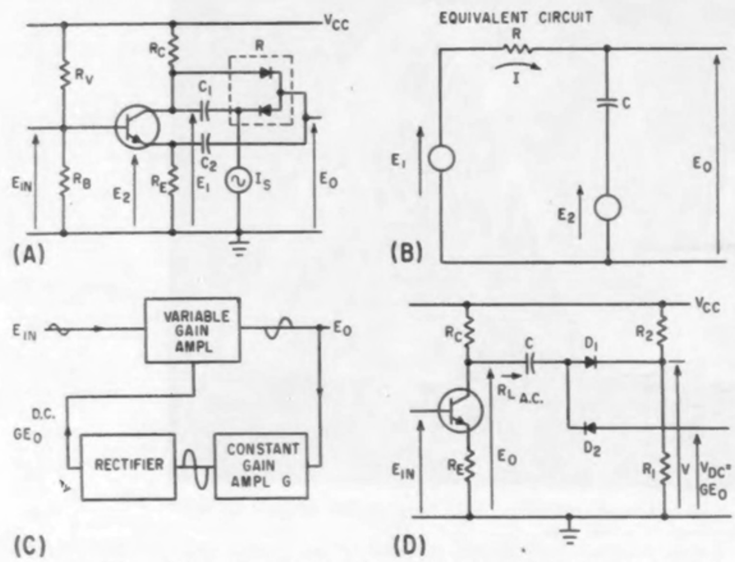

FIG. 1—Phase-shift circuit, (A); its equivalent circuit, (B); block diagram of automatic gain control circuit, (C); simplified schematic of the agc circuit, (D)

about 0.3 percent per degree Kelvin. Because of surface leakage current, Eq. 2 is not exact for the silicon diodes finally used, but it gives close to the correct magnitude of the temperature dependence. For constant-voltage bias, the ratio of junction currents at different temperatures is[4]

$$\frac{I_j}{I_{jo}} = \frac{T^2}{T_o}\exp\left[\frac{E_g - eV_o}{K}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right] \quad (3)$$

where I_{jo} is the current at T_o, I_j the current at T, E_g the energy gap of the material, V_o the externally applied junction voltage, e the charge of an electron, and k is Boltzmann's constant. At 300 degrees Kelvin, Eq. 3 indicates a change of about 15 percent per degree for silicon. Thus it is advantageous to bias the diodes in constant-current mode.

Although the control element could be incorporated with any of the conventional R-C phase-shift networks, such as the Wien bridge or the 3 or 4-section ladder network, a variation of an all-pass lattice network was chosen, as shown in Fig. 1A and 1B. The transfer function for this circuit, with $R_C = R_E$, $E_{in} = E_2 = -E_1$, $X_{C1} << R$, and a current source I_s of infinite impedance is

$$\frac{E_o}{E_i} = \frac{j\omega RC - 1}{j\omega RC + 1}$$

$$= \frac{[(\omega RC)^2 + 1]^{1/2}}{[(\omega RC)^2 + 1]^{1/2}} \angle 180^\circ - 2\tan^{-1}\omega RC \quad (4)$$

Thus when $\omega = 1/RC$

$$\frac{E_o}{E_i} = 1 \angle 90^\circ \quad (5)$$

With the above assumptions the circuit has unity gain at all frequencies and a phase shift that is a function of the product RC. For a constant phase shift of 90 deg through the network, the relationship $\omega = 1/RC$ must hold, and if C is held constant, the frequency at which 90 deg phase shift occurs is proportional to the reciprocal of the resistance. By cascading two of these networks with isolation, a phase shift of 180 deg is obtained at $\omega = 1/RC$. In the present application the resistance R is the controlled diode resistance, r_{a-c}, and therefore the frequency at which 180 deg phase shift occurs is proportional to the diode current. Figure 1A shows two diodes biased in series used as the control element. With the a-c coupling of C_2, however, the two diodes are in parallel. This arrangement is desirable, since despite the low-level signal amplitude the diode resistance is slightly nonlinear with applied signal. The use of two diodes presents a symmetrical resistance of the collector and hence reduces signal distortion.

Using the expression derived for ϕ_1 of the phase-shift network, the deviation in frequency from the predicted value due to extra phase shift in the coupling networks and additional circuits for two sections in cascade is

$$\phi_2 = 2\pi - 4\tan^{-1}\omega RC \quad (6)$$

and thus

$$\frac{d\omega}{d\phi_2} = \frac{-1 + (\omega RC)^2}{4RC} \quad (7)$$

which becomes, for $\omega = 1/RC$, $d\omega/d\phi_2 = -\omega/2$. Therefore, if the oscillator frequency is to be within 5 percent of the predicted value at a given bias current, the cumulative additional phase shift must be less than 5.7 degrees.

In deriving the transfer function of the phase shift network, simpli-

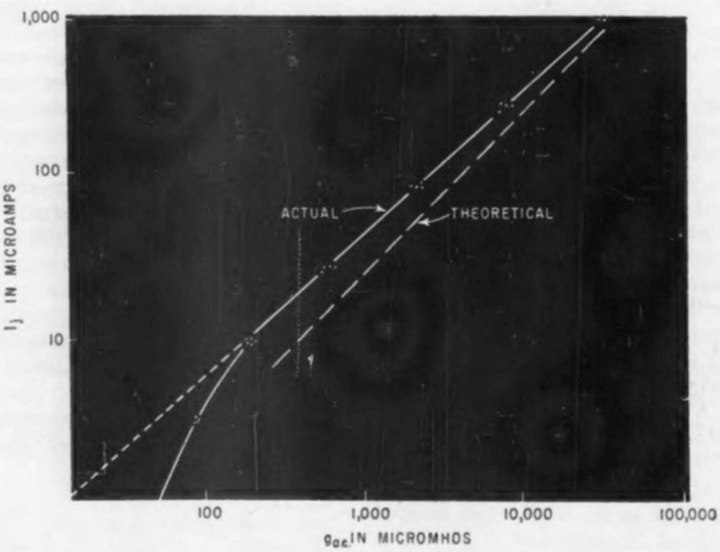

FIG. 2—Bias current plotted against junction resistance for 1N748 diodes

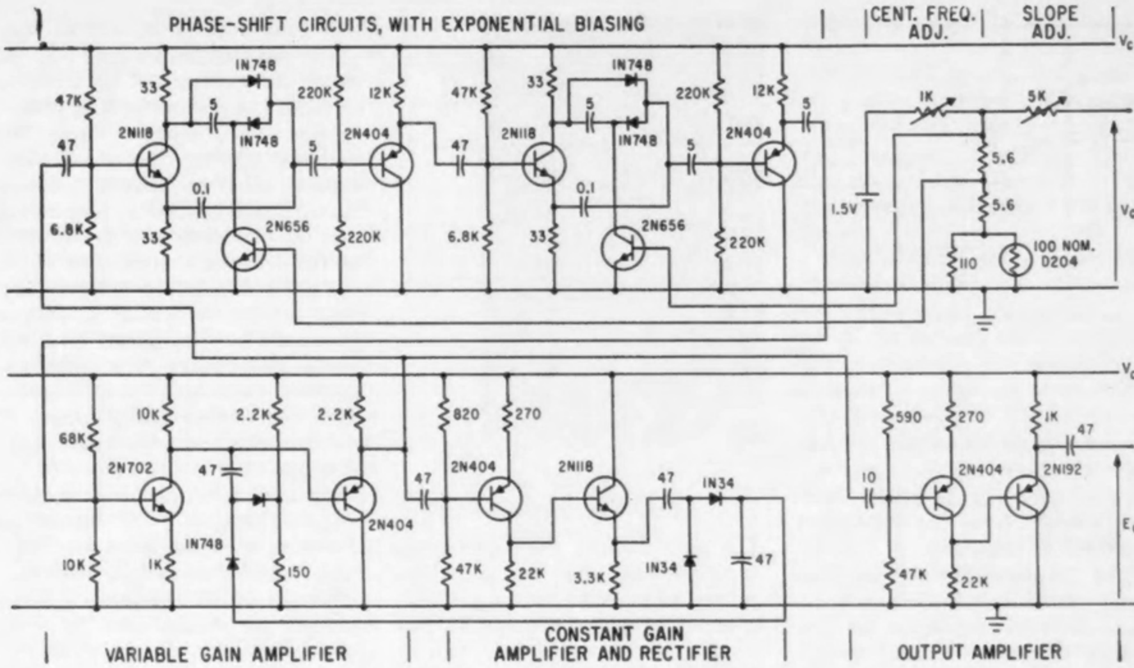

FIG. 3—Wide-range oscillator, with phase-shift circuit, upper left. Signal then goes to lower left

fying assumptions were made. The voltage gain of the phase inverter must be less than unity and hence for oscillation to occur, some additional voltage gain as well as another 180-deg phase shift must be introduced in closing the loop. The derivation also neglected the reduction of gain at high frequencies caused by the shunting effect of the internal capacitance associated with the transistors. Because capacitive coupling between sections is necessary to isolate the controlling bias current, there is also a reduction of gain at the low end of the frequency spectrum. Some form of automatic gain control is therefore necessary to keep the amplitude of oscillation constant at all frequencies.

The diode equation (Eq. 1) shows that constant current—hence constant r_{a-c}—is obtained in the presence of a d-c biasing voltage only when the amplitude of the input signal is less than kT/e. Since $kT/e = 0.025$ volt at room temperature, the a-c voltage should be much less than this for low distortion. Amplitude control is therefore necessary to maintain the diode peak-to-peak signal voltage in the low millivolt range. Without such control the oscillation would build up until the nonlinear characteristics of the amplifier or phase-shift network limited further increases in amplitude. Under the usual bias conditions, this would be a peak-to-peak voltage in volts, rather than millivolts, resulting in severe signal distortion.

The automatic gain control circuit consists of a variable-gain amplifier, a constant-gain amplifier to increase the magnitude sufficiently for rectification and control, and a rectifier that feeds back to the initial variable-gain amplifier. The load impedance and therefore the gain of the variable-gain amplifier is controlled by the rectifier output so that it provides a constant-amplitude signal. This is shown by analysis of the circuit of Fig. 1C and 1D. Assuming that $X_c << R_{La-c}$, $R_{La-c} << R_c$, $R_1 << R_{La-c}$, and $Z_o << R_{La-c}$

$$\frac{E_o}{E_{in}} \simeq \frac{R_{Lac}}{R_E} \tag{8}$$

But

$$R_{Lac} = \frac{kT}{2e}\,\frac{1}{I_o + I_j} \tag{9}$$

and

$$I_j = I_o\left[\exp\left(\frac{V_{dc} - V_R}{2kT/e}\right) - 1\right] \tag{10}$$

Also, if $V_{d-c} = G\,|E_o|$ and $|E_{in}| = F\,|E_o|$ where F is some positive fraction, $0 < F < 1$, these equations may be combined to obtain

$$E_o = \frac{2kT/e\left(\ln F + \ln \dfrac{kT/e}{2R_E I_o}\right) + V_R}{G} \tag{11}$$

Therefore, if

$$V_R + 2kT/e\left(\ln \frac{kT/e}{2R_E I_o}\right) \quad |(2kT/e)\ln F| \tag{12}$$

then

$$|E_o| \cong \frac{V_R + 2kT/e\left(\ln \dfrac{kT/e}{2R_E I_o}\right)}{G} \tag{13}$$

which is independent of variations in F. Therefore, the output voltage will remain constant despite variations in the gain F, of the phase-shift circuit.

Silicon 1N748 diodes were tested to determine the relationship between diode current and the equivalent small-signal diode resistance. Silicon diodes were chosen in preference to germanium diodes because of their smaller leakage current, which increases the linear range of r_{a-c} or g_{a-c} against current at low currents.

Measurements show that the diodes are consistent in their behavior, as can be seen from Fig. 2. An extension of the linear portion of the graph verifies the effect of leakage current. Although the data points

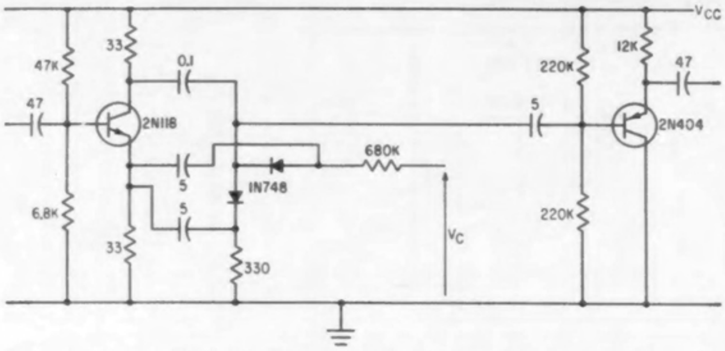

FIG. 4—Single section of phase-shift circuit with linear biasing

display a linearity on the logarithmic coordinates for about two decades of current, the slope differs considerably from the slope of 1 predicted by theory.

The consequences of the nonlinear relationship between conductance and bias current on the control of frequency depends on the control desired. Since the oscillating frequency is proportional to g_{a-c}, the frequency will bear this same power relationship to the control voltage when a current proportional to control voltage is used to bias the diodes. If, however, an exponential current source is used, such as a grounded emitter transistor, the power relationship serves only to vary the slope of the resulting V_b against frequency curve. For the grounded-emitter configuration, the base-voltage collector current characteristic can be approximated by

$$I_c \cong K_1 \exp (K_2 V_b) \qquad (14)$$

where K_1 and K_2 are constants. But $I_c = I_j$, for this biasing scheme, and if $g_{a-c} = A\, I^B$, where A and B are constants, then

$$\left(\frac{g_{a-c}}{A}\right)^{1/B} = \ln K_1 + K_2 V_b \qquad (15)$$

Then, since K_1 is a constant,

$$V_b \cong \frac{1}{K_2 B} \ln \left(\frac{g_{a-c}}{A K_1^{B}}\right) \qquad (16)$$

The basic form of the relationship is independent of B, and since frequency is proportional to g_{a-c}, the linear relationship between V_b and ln (frequency) still exists. This is a decisive advantage over the linear mode of biasing. For the transistors used, the proportionality between V_b and ln (I_c) holds for collector currents less than 100 microamp, but deviates from this relation at higher currents. Thus, for I_c greater than 100 microamp, it takes a larger V_b to produce a given I_c than would be predicted by the low current slope. This effect is due to the ohmic resistance of the base region. At collector current levels below 100 microamp, corresponding to bias currents less than 5 microamp, the effect of a base resistance of 200 ohms would produce a deviation of less than a millivolt, which for our application would be negligible. The 2N656 transistors were chosen as the current source in the oscillator because of their small base resistance and consequently small deviation from the desired relationship between V_b and ln (I_c).

The over-all schematic, Fig. 3, shows that emitter followers isolate amplifiers to prevent loading effects by succeeding stages. Capacitive coupling between the phase shift network and the emitter follower blocks the d-c biasing current so that the bias current is determined by the current source. The equal emitter and collector resistors are small in value to reduce the output impedance of the phase inverter. For a given range of variable resistance, the value of C will determine the corresponding range of frequency for which the phase shift is 180 degrees according to the relation $\omega = 1/r_{a-c}C$. For a desired range of frequency, C should be chosen to correspond to the optimum range of resistance. This optimum range of resistance is limited at the high end by the nonlinearity that occurs when the control current approaches the value of the leakage current. When a transistor is used as the control current source, the low end of the useful resistance control range is limited by the deviation of the transistor from an ideal exponential current source at higher currents.

Figure 5A shows a plot of the base voltage to produce a phase shift of 180 degrees against ln (frequency) for the phase shift circuit shown in Fig. 3. For this frequency range, the magnitude of phase shift due to capacitance coupling at low frequencies and transistor capacitance coupling at high frequencies is negligible, and the deviation from the linear portion of the graph at the low and high ends are due to the control limitations.

For the amplitude control circuit of Fig. 1D the output voltage will be approximately constant, inde-

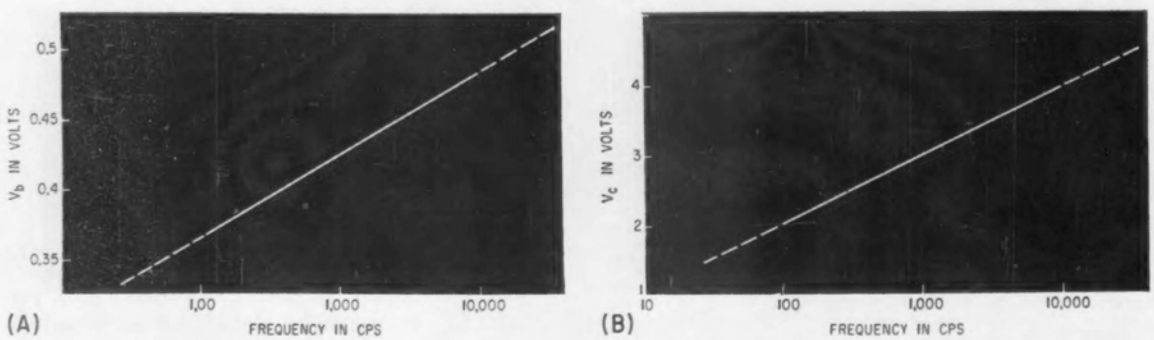

FIG. 5—Base voltage to produce a 180-deg phase shift plotted against frequency, (A); control voltage against oscillator frequency, (B), using exponential biasing

pendent of variations of gain of the phase shift network if Eq. (12) is satisfied. The magnitude of the output voltage will then be determined by V_B, R_B, the theoretical leakage current of the diodes D_1 and D_2, the gain constant G and the temperature of the diodes. For the values of components shown in Fig. 3, the predicted value of E_o is 0.0395 volt peak to peak, close to the measured value of 41 millivolts peak to peak. The effect of temperature on the amplitude is quite small for the values chosen, causing an increase in amplitude of about 2 percent for a 10 deg C increase in temperature. Substitution of the constants listed in Eq. (11) shows that the predicted variation in E_o is less than 7 percent for a change in the gain of the phase-shift section from 1 to 0.1.

The time constant of the voltage doubler type rectifier must be much greater than the period of the lowest frequency of oscillation to provide adequate filtering and d-c amplitude-control bias. Thus if the oscillator is to be frequency modulated by an a-c signal, the frequency of this modulating signal must be much less than the reciprocal of the rectifier time constant if amplitude modulation is to be minimized.

A series of tests to determine over-all performance were performed on the oscillator with exponential biasing as shown in Fig. 3. Figure 4 shows the modification in phase-shift circuits when linear biasing is used. The results of these tests are shown graphically in Fig. 5B through 7.

Figure 5B shows that the frequency of oscillation is variable over a range of 2 decades with about 5 percent deviation from a linear V_c versus ln (frequency) relationship. The deviation at frequencies above and below this range are due to the control limitations.

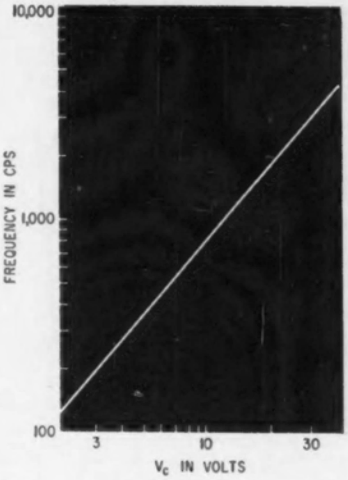

FIG. 6—Oscillator frequency vs control voltage, linear biasing

Figure 6 shows the data obtained for oscillator frequency against control voltage for the linear biasing network of Fig. 4. The measured slope of these data points is approximately 1.16. This is a direct consequence of g_{e-c} being proportional to the bias current raised to a power slightly different from unity. A range of more than two decades can be obtained with linear biasing.

The amplitude of the output voltage is constant to within about 2 percent for the frequency range of 100 to 10,000 cps as may be seen in Fig. 7A. This data was obtained using a d-c control voltage. At the upper and lower extremes of oscillation of 77,000 and 1.3 cps, the amplitude is about 1.1 volts peak to peak.

The total harmonic distortion of the output waveform at various frequencies from 100 to 10,000 cps was measured. For this range of frequencies, the distortion is low, the highest measurement being 3.2 percent.

Because of the strong dependence of the transistor collector current on temperature for constant base to emitter voltage, the frequency of the uncompensated oscillator is sensitive to ambient temperature. For the circuit of Fig. 3 without the thermistor compensation in the base to emitter circuit, the oscillator frequency increases by a factor of about 3 for a 10 deg C rise in ambient temperature. Figure 7B shows the temperature dependence of the thermistor compensated oscillator.

The linear biased oscillator with no compensation revealed an almost linear temperature dependence with a slope of about 2 percent per 10 deg C rise in temperature.

The oscillator displays a remarkable stability with respect to changes in supply voltage. Although the oscillator was designed for a V_{cc} of 15 volts, the supply voltage can vary over about a two to one range and the oscillator frequency will remain constant to within 2 percent at the lower frequencies and to within 10 percent at the higher frequencies. A two-volt deviation from 15 volts in supply voltage produces a frequency change of less than 1 percent at all frequencies from 100 to 10,000 cps.

For the case of the exponential biasing mode, a center frequency and slope adjustment are provided, as may be seen in Fig. 3. The use of a mercury cell provides a center frequency control that is independent of supply-voltage variations. The slope can be adjusted to yield 2 decades of frequency range with a change in control voltage as small as 0.2 volt or as great as 10 volts.

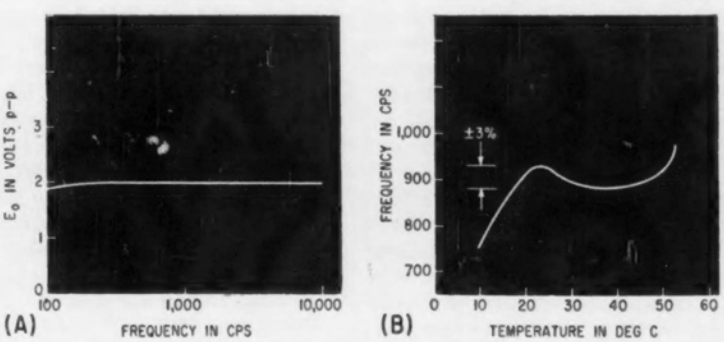

FIG. 7—Amplitude of output voltage is almost linear with oscillator frequency, (A); oscillator frequency against temperature for exponential biasing, (B)

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